Gluino-Mediated Rare B Decays [*]

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We discuss the gluino-induced contribution to rare B decays in supersymmetric frameworks with generic sources of flavour change.

1. Introduction

Apart from the low-energy regime of the strong interaction, flavour physics is the least tested part of the SM. This is reflected in the rather large error bars of several flavour parameters such as the mixing parameters at the twenty percent level [1]. However, the experimental situation concerning B physics will drastically change in the near future. There are several B physics experiments successfully running at the moment. In the upcoming years new facilities will start to explore B physics with increasing sensitivity and within different experimental settings [2].

The b quark system is an ideal laboratory for studying flavour physics. Hadrons containing a b quark are the heaviest hadrons experimentally accessible. Since the mass of the b quark is much larger than the QCD scale, the long-range strong interactions are expected to be comparably small and are well under control thanks to the heavy quark expansion [3,4].

Of particular interest are the so-called rare B decays, which are flavour changing neutral current processes (FCNC) which vanish at the tree level of the SM. Thus, they are rather sensitive probes for physics beyond the SM [5,6].

One of the main difficulties in analysing rare B decays is the calculation of short-distance QCD effects. These radiative corrections lead to a tremendous rate enhancement. The QCD radiative corrections bring in large logarithms of the form $\alpha_s^n(m_b) \log^m(m_b/M)$, where M is the top quark or the W mass and $m \leq n$ (with $n = 0, 1, 2, ...$). They have to get resummed at least to leading-log (LL) precision ($m = n$).

Within the SM the accuracy in the dominating perturbative contribution to $B \to X_s\gamma$ was recently improved to NLL precision. This was a joint effort of many different groups [7]. The theoretical error of the previous leading-log (LL) result was substantially reduced to $\pm 10\%$ and the central value of the partonic decay rate increased by about 20%.

Supersymmetric extensions of the SM have become the most popular framework of new theoretical structures at higher scales, much below the Planck scale. The precise mechanism of the necessary supersymmetry breaking is unknown [8]. A reasonable approach to this problem is the inclusion of the most general soft breaking term consistent with the SM gauge symmetries in the so-called unconstrained minimal supersymmetric standard model (MSSM). This leads to a proliferation of free parameters in the theory.

A global fit to electroweak precision measurements within supersymmetric models shows that if the superpartner spectrum becomes light the fit to the data results in typically larger values of χ^2 compared with the SM. Supersymmetric models, however, can always avoid serious constraints from data because the supersymmetric contributions decouple [9].

In the MSSM there are two kinds of new contributions to FCNC processes. The first class results from flavour mixing in the sfermion mass matrices [10]. Moreover, one has CKM-induced contributions from charged Higgs boson and chargino exchanges (see [11]). This leads to the well-known supersymmetric flavour problem: the severe experimental constraints on flavour violations have no direct explanation in the structure of the unconstrained MSSM. Clearly, the origin of flavour violation is a model-dependent issue and is based on the relation of the dynamics of flavour and the mechanism of supersymmetry breaking. Keeping in mind our current phenomenological knowledge about supersymmetry, it is suggestive to perform a model-independent analysis of flavour changing phenomena. Such an analysis provides important hints on the more fundamental theory of soft supersymmetry breaking.

2. Gluino Contribution to $B \to X_s \gamma$

Among inclusive rare B decays, the $B \to X_s \gamma$ mode is the most prominent because it is the only decay mode in this class that is already measured [12,13]. Many papers are devoted to studying the $B \to X_s \gamma$ decay and similar decays within the MSSM. However, in most of these analyses, the contributions of supersymmetry were not investigated with the systematics of the SM calculations. In [14] it was shown, that in a specific supersymmetric scenario NLL contributions are important and lead to a significant reduction of the stop-chargino mass region where the supersymmetric contribution has a large destructive interference with the charged-Higgs boson contribution. It is expected that the complete NLL calculation drastically decreases the scale dependence and, thus, the theoretical error. The NLL analysis is also a necessary check of the validity of the perturbative ansatz (see [15]). The NLL calculations in [14] and also in [16] are worked out in the heavy gluino case. In the analysis [17] reported here, the gluino-mediated decay $B \to X_s \gamma$ is discussed where the gluino is not assumed to be decoupled. Previous work on the gluino contribution [10, 18,19] did not include LL or NLL QCD corrections, and gluino exchanges were treated in

the so-called mass insertion approximation (MIA) only, where the off-diagonal squark mass matrix elements are taken to be small and their higher powers neglected. In our analysis we explore the limits of the MIA. Furthermore, we analyse the sensitivity of the bounds on the sfermion mass matrices to radiative QCD corrections.

Within the SM, there is one coupling constant, G_F, relevant to the $b \to s\gamma$ decay. There is also one flavour violation parameter only, namely the product of two CKM matrices. All the loops giving the logarithms are due to gluons, which imply a factor of α_s. The corrections can then be classified according to:

- (LL), $G_F(\alpha_s Log)^N$,

- (NLL), $G_F \alpha_s (\alpha_s Log)^N$.

Thus, the above ordering also reflects the actual size of the contributions to $b \to s\gamma$.

The corresponding analysis of QCD corrections in the MSSM is much more complicated. The MSSM has several couplings relevant to this decay and there are several flavour changing parameters. Thus, a formal LL term might have a small coupling while a NLL contribution is multiplied with a large one. Moreover, the couplings generally depend on the parameters, and the results should be applicable for large domains on the parameters.

Another complication in supersymmetric theories is the occurrence of flavour violations such as gluino exchanges (through the gluino-quark-squark coupling) where additional factors α_s are induced. They lead to magnetic penguin operators where the Wilson coefficients naturally contain factors of α_s. Moreover, these contributions induce magnetic operators where the (small) factor m_b is replaced by the gluino mass. Clearly this contribution is expected to be dominating. The gluino-induced contributions to the decay amplitude for $b \to s\gamma$ are of the following form:

- (LL), $\alpha_s (\alpha_s Log)^N$

- (NLL), $\alpha_s \alpha_s (\alpha_s Log)^N$

In the matching calculation, all factors α_s regardless of their source should get expressed in

terms of the α_s running with five flavours. However, non-decoupling effects through violations of the supersymmetric equivalence between gauge bosons and corresponding gaugino couplings have to be taken into account at the NLL level.

Furthermore, one finds that gluino-squark boxes induce new scalar and tensorial four-Fermi operators, which are shown to mix into the magnetic operators without gluons. On the other hand, the vectorial four-Fermi operators mix only with an additional gluon into magnetic ones. Thus, they will contribute at the next-to-leading order only. However, from the numerical point of view the contributions of the vectorial operators (although NLL) are not necessarily suppressed w.r.t the new four-Fermi contributions; this is due to the expectation that the flavour-violation parameters present in the Wilson coefficients of the new operators are expected to be much smaller (or much more stringently constrained) than the corresponding ones in the coefficients of the vectorial operators. This is one of the most important reasons why a complete NLL order calculation should be performed.

The mixed graphs, containing a W, a gluino and a squark, are proportional to $G_F \alpha_s$. They give rise only to corrections of the SM operators at the NLL level. There are also penguin contributions with two gluino lines in the NLL matching.

The current discussion is restricted to the W or gluino-mediated flavour changes and does not consider contributions with other Susy particles such as chargino, charged Higgs or neutralino. Clearly, analogous phenomena occur in those contributions.

To understand the sources of flavour violation that may be present in supersymmetric models in addition to those enclosed in the CKM matrix, one has to consider the contributions to the mass matrix of a squark of flavour f:

$$\mathcal{M}_f^2 = \begin{pmatrix} m_{f,LL}^2 & m_{f,LR}^2 \\ m_{f,RL}^2 & m_{f,RR}^2 \end{pmatrix} + \qquad (1)$$

$$+ \begin{pmatrix} F_{f,LL} + D_{f,LL} & F_{f,LR} \\ F_{f,RL} & F_{f,RR} + D_{f,RR} \end{pmatrix} \qquad (2)$$

In the super CKM basis where the quark mass matrix is diagonal and the squarks are rotated in parallel to their superpartners, the F terms from the superpotential and the D terms in the 6×6 mass matrices $\mathcal{M}_f^2$ turn out to be diagonal 3×3 submatrices. This is in general not true for the additional terms (1) from the soft supersymmetry breaking potential. Because all neutral gaugino couplings are flavour diagonal in the super CKM basis, the gluino contributions to the decay width of $b \to s\gamma$ are induced by the off-diagonal elements of the soft terms $m_{f,LL}^2$, $m_{f,RR}^2$, $m_{f,RL}^2$.

3. Numerical Results

We show a few features of our numerical results based on a complete LL calculation. More details of the analysis can be found in [17]. The size of the gluino contribution crucially depends on the soft terms in the squark mass matrix $\mathcal{M}_{\tilde{D}}^2$ and to a lesser extent on those in $\mathcal{M}_{\tilde{U}}^2$. In the following, we take all the diagonal entries in the soft matrices $m_{\tilde{Q},LL}^2$, $m_{\tilde{D},RR}^2$, $m_{\tilde{U},RR}^2$, to be equal; their common mass is denoted by $m_{\tilde{q}}$ and set to the value 500 GeV. First, the matrix element $m_{\tilde{D},LR;23}^2$ is varied. All other entries in the soft mass terms are put to zero. Following the notation of [19], we define

$$\delta_{LR;23} = m_{\tilde{D},LR;23}^2 / m_{\tilde{q}}^2 \quad \text{and} \quad x = m_{\tilde{g}}^2 / m_{\tilde{q}}^2, \quad (3)$$

where $m_{\tilde{g}}$ is the gluino mass. In Fig. 1, the QCD-corrected branching ratio is shown as a function of x (solid lines), obtained when only $\delta_{LR,23}$ is vanishing ($\delta_{LR,23} = 0.01$). Shown is also the range of variation of the branching ratio, delimited by dotted lines, obtained when the low-energy scale μ_b spans the interval 2.4–9.6 GeV. The matching scale μ_W is here fixed to m_W. As can be seen, the theoretical estimate of $BR(B \to X_s\gamma)$ is still largely uncertain ($\sim \pm 25\%$). An extraction of bounds on the δ quantities more precise than just an order of magnitude, therefore, would require the inclusion of next-to-leading logarithmic QCD corrections. It should be noticed, however, that the inclusion of the LL QCD corrections has already removed the large ambiguity on the value to be assigned to the factor $\alpha_s(\mu)$ in the gluino-induced operators. Before adding QCD corrections, it is not clear
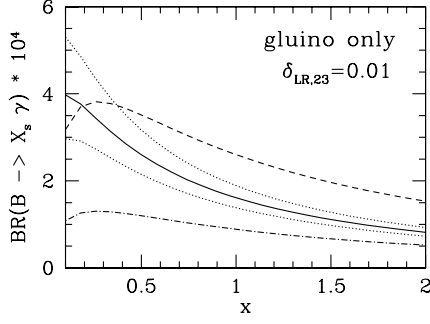
Figure 1. Gluino-induced branching ratio $BR(B \to X_s\gamma)$ as a function of $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$, when only $\delta_{LR,23}$ is non-vanishing (see text).

Figure 3. Mass insertion approximation (dashed line) vs. exact result (solid line) as a function of $\delta_{LL;23}$ (see text).
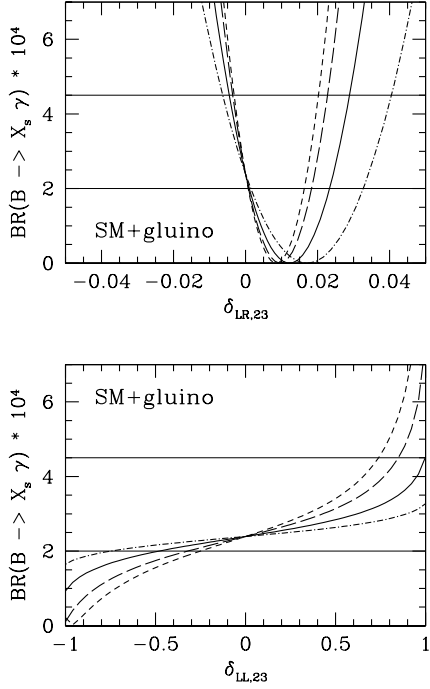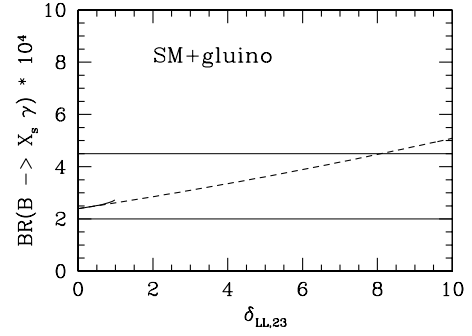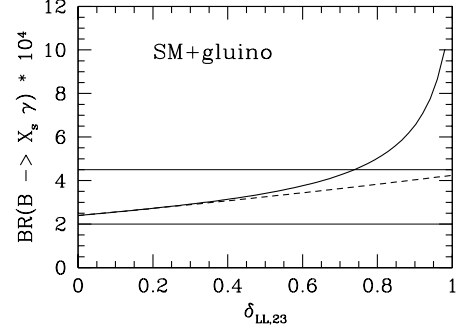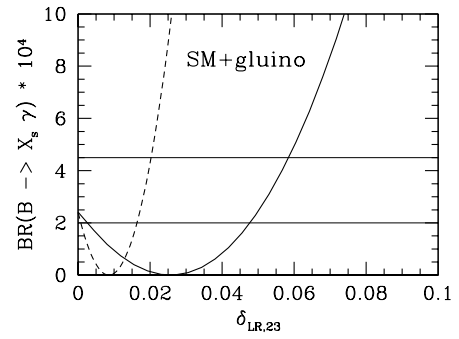

Figure 2. $BR(B \to X_s\gamma)$ with $x = 0.3$ (short-dashed line), 0.5 (long-dashed line), 1 (solid line), 2 (dot-dashed line), see text.

Figure 4. $BR(B \to X_s\gamma)$ as a function of $\delta_{LR;23}$ including interference effects with a chain $\delta_{LL;23}\delta_{LR;33}$ in solid line (see text).

whether the explicit α_s factor should be taken at some high scale μ_W or a some low scale μ_b, the difference is a LL effect. The corresponding values for $BR(B \rightarrow X_s\gamma)$ for the two extreme choices of μ are indicated in Fig. 1 by the dot-dashed lines ($\mu = m_W$) and the dashed lines ($\mu = 4.8\,\mathrm{GeV}$). The branching ratio is then virtually unknown.

In spite of the large uncertainties which the branching ratio $BR(B \rightarrow X_s\gamma)$ still has at the LL in QCD, it is possible to extract indications on the size that the δ-quantities may maximum acquire without inducing conflicts with the experimental measurements. As already noted in Ref. [19], the element $\delta_{LR,23}$ is certainly the flavour-violating parameter most efficiently constrained. In Fig. 2 , the dependence of $BR(B \rightarrow X_s\gamma)$ is shown as a function of this parameter when this is the only flavour-violating source. The branching ratio is obtained by adding the SM and the gluino contribution obtained for different choices of x for the fixed values $\mu_b = 4.8\,\mathrm{GeV}$ and $\mu_W = m_W$. The gluino contribution interferes constructively with the SM for negative values of $\delta_{LR,23}$, which are then more sharply constrained than the positive values. Overall, this parameter cannot exceed the per cent level. Much weaker is the dependence on $\delta_{LL,23}$ if this is the only off-diagonal element in the down squark mass matrix. This dependence is illustrated in Fig. 2 for different choices of x. The induced gluino contribution interferes constructively with the SM contribution for positive $\delta_{LL,23}$. Notice that given the large values of $\delta_{LL,23}$ allowed by the experimental measurement, the MIA cannot be used in this case to obtain a reliable estimate of $BR(B \rightarrow X_s\gamma)$, whereas it is an excellent approximation of the complete calculation in the case of $\delta_{LR,23}$. In the upper frame of Fig. 3, we vary $\delta_{LL;23}$ for $x = 0.3$. The MIA and the exact result start to deviate considerably for $x > 0.4$, i.e. well within the experimental error band; the exact result leads to more stringent bounds. An even more drastic example is shown in the lower frame of Fig. 3, where we increase $m_{\tilde{g}}$ in such a way that $x = 4$, leaving all the other parameters unchanged: for $\delta_{LL;23} > 0.1$ the mass matrix $\mathcal{M}_{\tilde{D}}^2$ has at least one negative eigenvalue. This feature is of course completely missed in the

MIA. One also has to consider interference effects. In Fig. 4 we show that the additional contribution through a chain $\delta_{LL;23}\delta_{LR;33}$ weakens the bound on the parameter $\delta_{LR;23}$ significantly. In the solid curve we put $\delta_{LL;23} = \delta_{LR;33} = \sqrt{\delta_{LR;23}}$, while in the dashed curve $\delta_{LL;23} = \delta_{LR;33} = 0$. We have chosen again $x = 0.3$.

Finally, we stress that a consistent precision analysis of the bounds on the sfermion mass matrix should include a NLL calculation and also interference effects with the chargino contribution.

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